Dimension as an Invariant of Street Networks

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1 Introduction

Street networks have been examined in respect to their structure [1,2,3,4]. The author of this paper has previously examined networks from various domains, thereby demonstrating that the polynomial volume law applies to many of them [5]. This paper focuses on the geographical domain only. Thereby, it examines the dimension of street networks, which turns out to be very stable.

2 Networks and Space

Things exist and happen in space and time. While different conceptualizations of space exist – space as a container, space instantiated by objects and their relations [6], etc. – they all agree on the fact that space can provide structure to things that are placed in space. Mocnik [7,8] has discussed the influence of space on networks inheriting a spatial structure. Thereby, networks with a spatial structure, or spatial networks in short, can be regarded as a prototypical example of spatial information that consists of relations.

The question of which properties are inherited from space depends on its conceptualization – Euclidean and metric space, topological space, or geographical space, to name a few. Tobler's law is an example of how space and relations mutually influence: 'everything is related to everything else, but near things are more related than distant things' [9]. When being interpreted in a geographical context, the law describes geographical information as being continuous. Tobler's law can prototypically be modelled by the Mocnik (network) model [5,7,8]. The model introduces edges between near nodes for a given set of nodes. Thereby, a node n_1 being 'near' to another node n_2 is defined as

$$\mathrm{dist}(n_1, n_2) \leq \rho \cdot \min_{m \neq n_1} \mathrm{dist}(n_1, m)$$

for some given constant $\rho > 1$. The resulting model has been shown to resemble many spatial networks, among them, street and public transport networks [5].

3 The Polynomial Volume Law

One of the most prominent characteristics of space is the possibility to measure volumes. In particular, the volume of a ball scales as r^d in dependence of the radius r, thereby encoding the dimension d. A concept similar to the volume of a ball can be

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introduced for thematic information represented by a network. The ball $B_n(r)$ can be defined as all nodes that are within network distance r from the centre node n. The volume of this ball is, in turn, defined as the number of the contained nodes.

When relations are influenced by space, it can be hypothesized that the volume of a ball in the corresponding network scales in the same way as the volume of a ball in space, called the *polynomial volume law* [5]. This law has turned out to be valid for a large number of networks, indicating that they are, in fact, strongly influenced by space. The volumes in a network can statistically be examined (for various centre nodes and radii) and fitted by the polynomial volume law. The resulting estimation of the parameter d can be regarded as the dimension of the network. This dimension by the polynomial volume law has been compared to other concepts of dimension in a network, exposing strong similarities and some differences [5]. Examples of brain, public transport, and road networks, among others, have been shown to follow this law with reasonable estimates of the dimension (Figure 1).

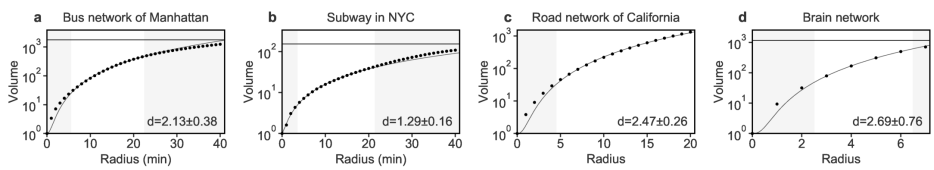

Fig. 1. Polynomial volume law at the examples of real-world networks. (a) Bus network of Manhattan; (b) Subway in New York City; (c) Road network of California; and (d) Brain network. The figure has been adapted from a previous publication [5] under the CC BY 4.0 license.

4 Dimension as an Invariant of Street Networks

While several real networks have been shown to follow the polynomial volume law, it has not yet been examined whether the estimate of the dimension is stable across many real-world networks of similar type to which they can thus serve as an invariant. Table 1 shows the dimension for a number of street networks. In fact, the dimension is very similar for these networks. The estimates are distributed around 2, which suggests itself due to the two-dimensional surface of the Earth. This is despite the varying size of the examined cities and their different network structures. In particular, the networks differ strongly in their centralization, which measures the variance of the centrality within the network, and their assortativity, given as the Pearson correlation coefficient of the degrees.

5 Conclusion

The influence of space on networks can not only be traced but is seemingly also very stable, as is demonstrated here at the example of street networks. Future research might examine further examples of networks and test for which kind of networks the dimension can serve as an invariant.

Table 1. Street networks. The networks are extracted from OpenStreetMap (http://openstreetmap.org) using OSMnx (https://github.com/gboeing/osmnx).

City	Dimension	Nodes	Edges	Centralization		Assortativity
				Degree	Betweenness	
Heidelberg, Germany	1.91 ± 0.51	2726	6403	$9.74 \cdot 10^{-4}$	$1.43 \cdot 10^{-1}$	$1.76 \cdot 10^{-1}$
Oxford, UK	1.92 ± 0.57	3363	7512	$5.26 \cdot 10^{-4}$	$1.77 \cdot 10^{-1}$	$-4.56 \cdot 10^{-2}$
Reykjavík, Iceland	1.93 ± 0.58	10157	20877	$1.92 \cdot 10^{-4}$	$1.96 \cdot 10^{-1}$	$-3.87 \cdot 10^{-2}$
Lund, Sweden	1.93 ± 0.60	1792	4298	$8.95 \cdot 10^{-4}$	$1.78 \cdot 10^{-1}$	$1.88 \cdot 10^{-1}$
Greenwich, UK	1.94 ± 0.59	4750	10891	$5.70 \cdot 10^{-4}$	$2.71 \cdot 10^{-1}$	$3.65 \cdot 10^{-2}$
Cambridge, UK	1.96 ± 0.60	3345	7360	$5.39 \cdot 10^{-4}$	$2.61 \cdot 10^{-1}$	$-1.00 \cdot 10^{-1}$
Melbourne, Australia	1.98 ± 0.53	180971	406940	$2.07 \cdot 10^{-5}$	$2.14 \cdot 10^{-1}$	$1.59 \cdot 10^{-1}$
Uppsala, Sweden	1.98 ± 0.57	8116	18931	$3.29 \cdot 10^{-4}$	$1.31 \cdot 10^{-1}$	$-1.86 \cdot 10^{-2}$
Bonn, Germany	1.99 ± 0.51	5827	13871	$4.50 \cdot 10^{-4}$	$2.21 \cdot 10^{-1}$	$7.37 \cdot 10^{-2}$
Bremen, Germany	1.99 ± 0.53	8903	20346	$3.05 \cdot 10^{-4}$	$2.44 \cdot 10^{-1}$	$1.09 \cdot 10^{-1}$
Greater London, UK	2.03 ± 0.54	124665	295088	$2.11 \cdot 10^{-5}$	$1.27 \cdot 10^{-1}$	$4.37 \cdot 10^{-2}$
Santa Fe, NM, USA	2.03 ± 0.57	4083	9895	$6.32 \cdot 10^{-4}$	$1.60 \cdot 10^{-1}$	$8.33 \cdot 10^{-2}$
Vienna, Austria	2.06 ± 0.49	16054	36183	$1.71 \cdot 10^{-4}$	$1.62 \cdot 10^{-1}$	$1.86 \cdot 10^{-1}$
Manhattan, NY, USA	2.07 ± 0.38	4473	9729	$8.56 \cdot 10^{-4}$	$2.00 \cdot 10^{-1}$	$3.65 \cdot 10^{-1}$
Albuquerque, NM, USA	2.08 ± 0.54	22537	57020	$1.10 \cdot 10^{-4}$	$2.65 \cdot 10^{-1}$	$2.05 \cdot 10^{-1}$
Münster, Germany	2.13 ± 0.55	6723	15878	$3.93 \cdot 10^{-4}$	$1.81 \cdot 10^{-1}$	$-1.83 \cdot 10^{-2}$
Paris, France	2.17 ± 0.51	10176	19947	$3.97 \cdot 10^{-4}$	$8.13 \cdot 10^{-2}$	$2.15 \cdot 10^{-1}$

Software. An implementation of the Mocnik model as well as of the algorithm to estimate the dimension of a network is published as part of NetworKit (https://github.com/kit-parco/networkit), an open-source toolkit for large-scale network analysis.

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